Benha University
Faculty Of Engineering at Shoubra

ECE 411
Antennas & Wave propagations
(2016/2017)

Lecture (5)
Array of Point Sources

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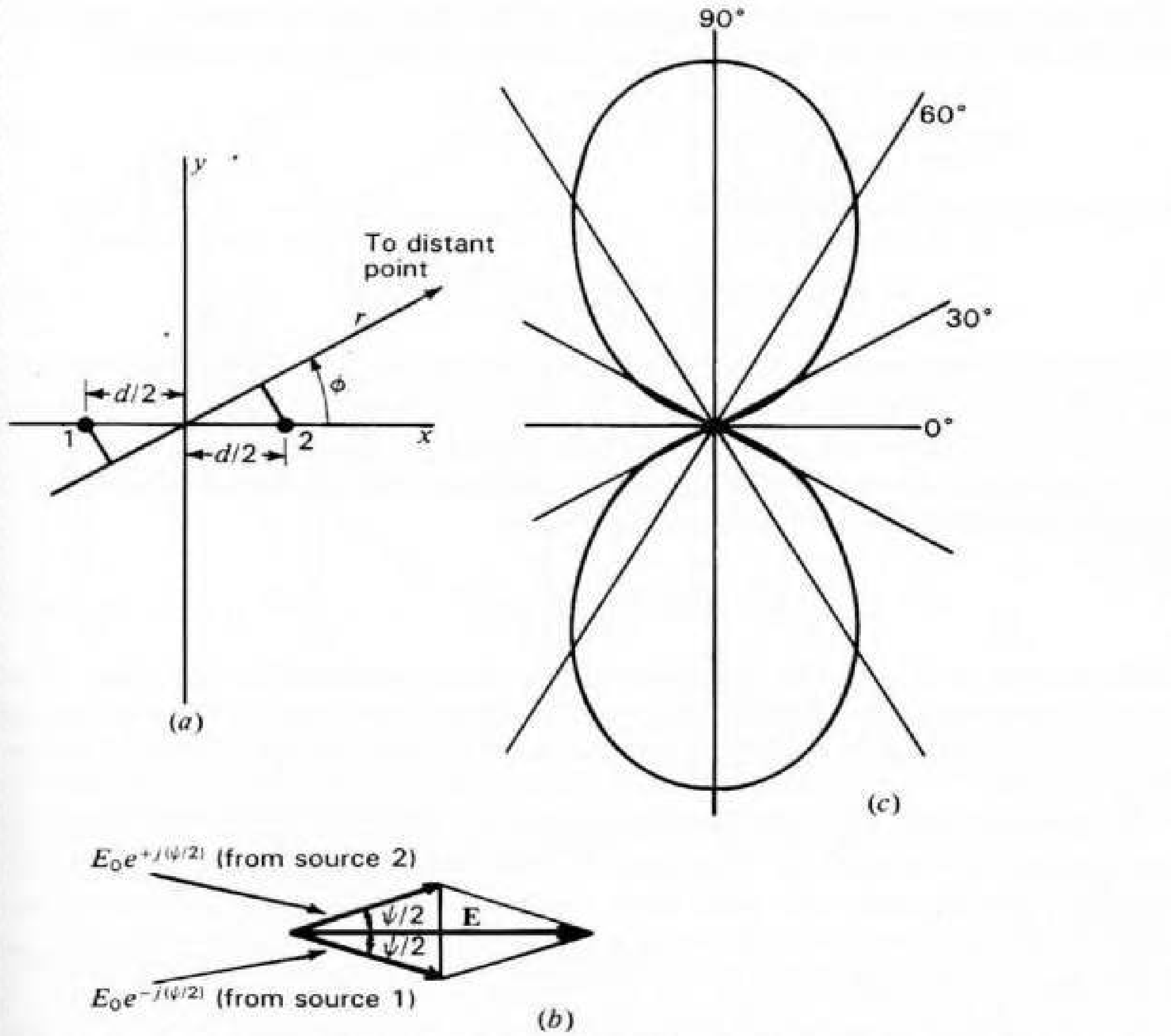
Agenda

- **1 - Two isotropic point sources (Cases)**
  - **Same amplitude and phase**
  - **Same amplitude and opposite phase**
  - **Same amplitude and phase Quadrature**
  - **Same amplitude and any phase**
  - **Unequal amplitude and any phase**
- **2 - Pattern Multiplication**

Two Isotropic Point Sources

Case 1 : 2 Sources with Same amplitude and phase



Case 1 : 2 Sources with Same amplitude and phase

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

The total field at a large distance r in the direction ϕ is then

$$E = E_0 e^{-j\psi/2} + E_0 e^{+j\psi/2}$$

where $\psi = d_r \cos \phi$ and the amplitude of the field components at the distance r is given by E_0.

$$E = 2E_0 \frac{e^{+j\psi/2} + e^{-j\psi/2}}{2}$$

$$E = 2E_0 \cos \frac{\psi}{2} = 2E_0 \cos \left(\frac{d_r}{2} \cos \phi \right)$$

set $2E_0 = 1$ d is $\lambda/2$ $d_r = \pi$.

Then $$E = \cos \left(\frac{\pi}{2} \cos \phi \right)$$

Another way for Estimation

The same pattern can also be obtained by locating source 1 at the origin and source 2 at a distance d along the positive x axis

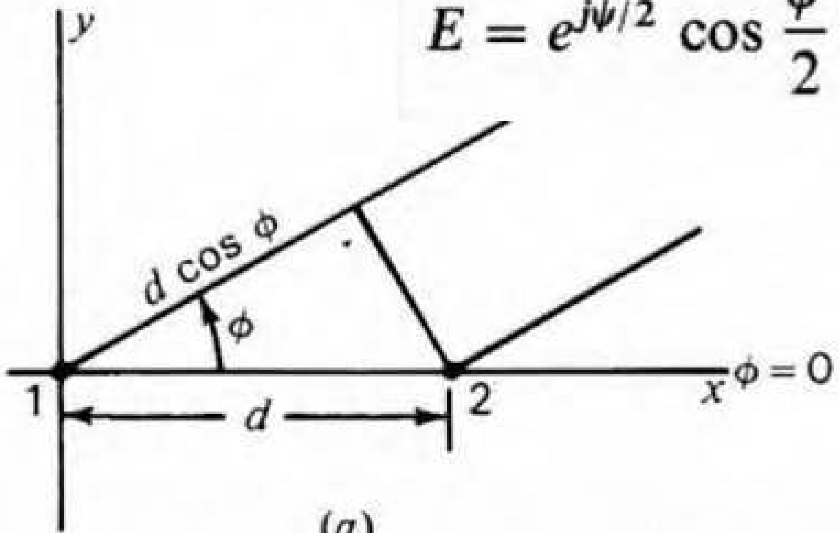
Taking now the field from source 1 as reference

Thus, the total field E $\quad E = E_0 + E_0 e^{+j\psi}$ $\quad$ where $\psi = d_r \cos\phi$

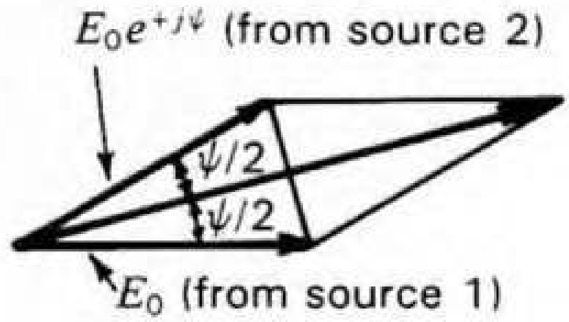
$$E = E_0(1 + e^{j\psi}) = 2E_0 e^{j\psi/2}\left(\frac{e^{j\psi/2} + e^{-j\psi/2}}{2}\right) = 2E_0 e^{j\psi/2} \cos\frac{\psi}{2}$$

Normalizing by setting $2E_0 = 1$

$$E = e^{j\psi/2} \cos\frac{\psi}{2} = \cos\frac{\psi}{2} \underline{/\psi/2}$$



(a)



(b)

Example (1)

ex:- for $d = \frac{\lambda}{2}$ draw the pattern of previous case

* **max. directions :-**

at $E_n = \pm 1 \Rightarrow \cos\left(\frac{\pi d}{\lambda}\cos\theta\right) = \pm 1$

$\therefore \frac{\pi d}{\lambda}\cos\theta = \pm k\pi$ or $\frac{\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\theta = \pm k\pi$

$\therefore \cos\theta = \pm 2k \quad (k = 0, 1, 2, \ldots)$

for $k = 0$ $\therefore \cos\theta = 0$ or $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

at $k =$ any value except $k = 0$ refused

$\therefore \boxed{\theta_{max} = 90^\circ \,\&\, 270^\circ}$

Example (1)

★ **nulls**

at $E_n = 0$ $\qquad \cos\left(\frac{\pi d}{\lambda}\cos\theta\right)$ or $\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$

$\therefore \quad \frac{\pi}{2}\cos\theta = \pm(2k+1)\frac{\pi}{2}$ → ٩٠° ومضاعفاتها الفردية

$\cos\theta = \pm(2k+1)$ $\qquad$ at $k = 0, 1, 2 \ldots$

for $k = 0$ $\qquad \cos\theta = \pm 1$ or $\theta = 0, 180$

k = any value (refused)

$$\therefore \quad \theta_{nulls} = 0^\circ, 180^\circ$$

Example (1)

* **Half Power directions**

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}} \qquad \therefore \frac{\pi}{2}\cos\theta = \pm(2k+1)\frac{\pi}{4}$$

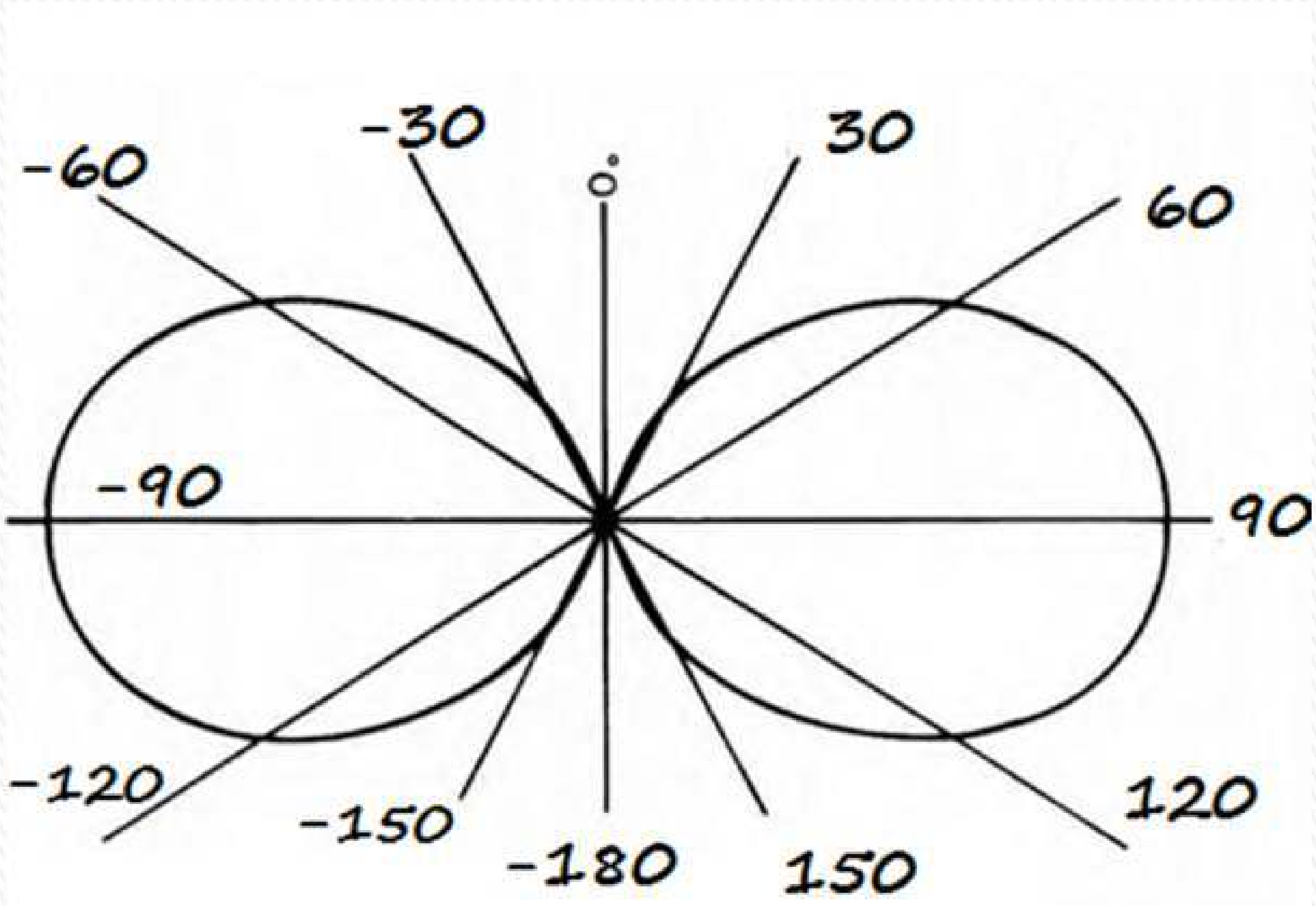
من مضاعفات الفردية

$$\cos\theta = \pm\frac{(2k+1)}{2} = \pm\left(k+\frac{1}{2}\right) \qquad k = 0, 1, 2, \ldots$$

$k = 0$ (only) $\quad \therefore \cos\theta = \pm\frac{1}{2} \quad \therefore \cos\theta = \begin{cases} \frac{1}{2} & (\pm 60^\circ) \\ -\frac{1}{2} & (\pm 120^\circ) \end{cases}$

$$\therefore \boxed{\theta = \pm 60^\circ, \pm 120^\circ}$$

Example (1)



Case 2 : Same amplitude and opposite phase

$$E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2}$$

$$E = 2jE_0 \sin\frac{\psi}{2} = 2jE_0 \sin\left(\frac{d_r}{2}\cos\phi\right)$$

$d = \lambda/2$,

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right)$$

The directions ϕ_m of maximum field are obtained by setting the argument

$$\frac{\pi}{2}\cos\phi_m = \pm(2k+1)\frac{\pi}{2}$$

The null directions ϕ_0 are given by $\frac{\pi}{2}\cos\phi_0 = \pm k\pi$

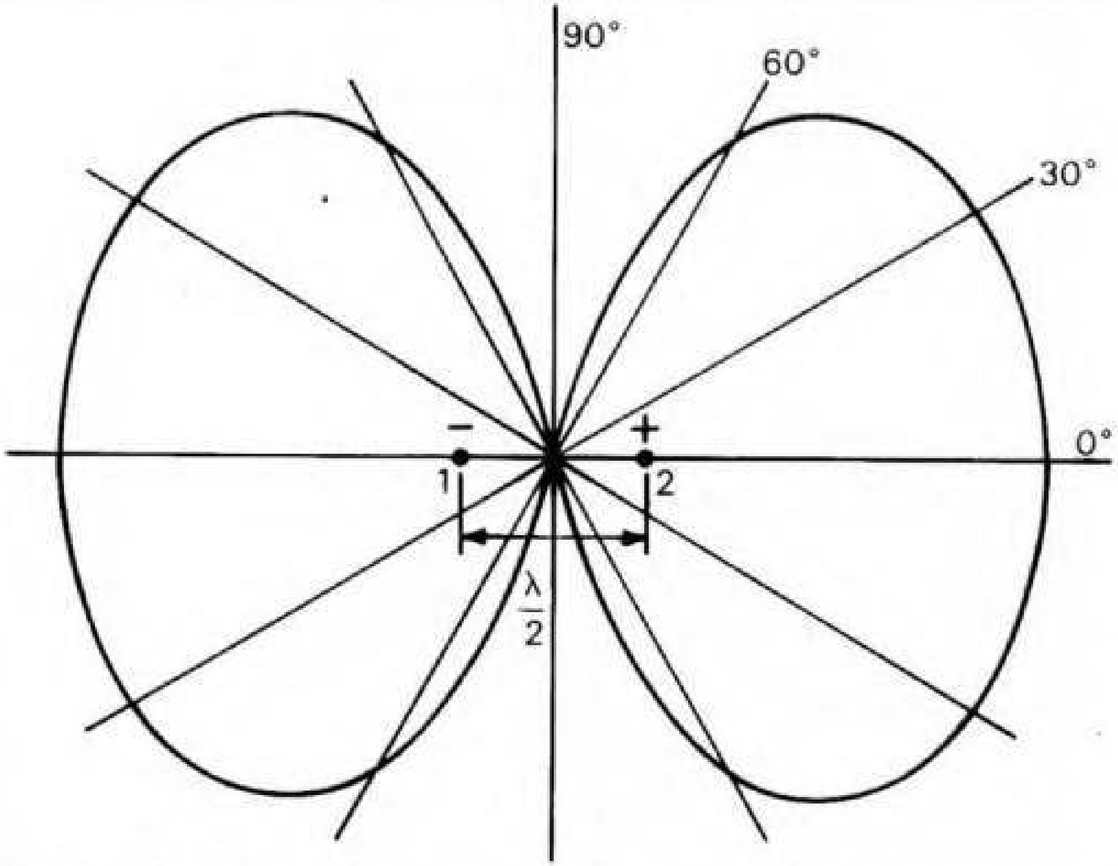
The half-power directions are given by $\frac{\pi}{2}\cos\phi = \pm(2k+1)\frac{\pi}{4}$

Case 2 : Same amplitude and opposite phase

Maximum $\phi_m = \pm 1$ and $\phi_m = 0^\circ$ and 180°.

Nulls $\phi_0 = \pm 90^\circ$.

Half Power $\phi = \pm 60^\circ, \pm 120^\circ$.



Case 3 : Same amplitude and phase Quadrature

$$E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right]$$

$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right)$$

Letting $2E_0 = 1$ and $d = \lambda/2$, (13) becomes

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$$

Example (2)

$d = \lambda/2$

d.i.e The total field pattern of 2 isotropic point sources of same amplitude but in phase quadrature:

i) max.

$$\cos\left(\frac{\pi}{2}\cos\varphi + \frac{\pi}{4}\right) = ??$$

$\swarrow$ ± 1

$$\frac{\pi}{2}\cos\varphi + \frac{\pi}{4} = 0$$

$$\frac{\pi}{2}\cos\varphi = -\frac{\pi}{4}$$

$$\frac{\pi}{2}\cos\varphi + \frac{\pi}{4} = \pm k\pi$$

$$\frac{\pi}{2}\cos\varphi = 0 - \frac{\pi}{4}$$

$$\cos\varphi = -(\pi/4 \div \pi/2)$$

$k = 0$

$$\varphi_m = \pm 120^\circ$$

$$\varphi_{max} = \pm 120^\circ$$

Example (2)

$d = \lambda/2$

ii) nulls

$$\cos\left(\frac{\pi}{2}\cos\varphi + \frac{\pi}{4}\right) = 0$$

$$\frac{\pi}{2}\cos\varphi + \frac{\pi}{4} = \pm(2K+1)\frac{\pi}{2}$$

$\boxed{K=0}$

$$\frac{\pi}{2}\cos\varphi + \frac{\pi}{4} = \pm\frac{\pi}{2}$$

$$\frac{\pi}{2}\cos\varphi = +\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \Rightarrow \cos\varphi = \frac{\pi}{4} \div \frac{\pi}{2}$$

$$\varphi = \cos^{-1} 0.5 = 60^\circ$$

$$\frac{\pi}{2}\cos\varphi = -\frac{\pi}{2} - \frac{\pi}{4} = -\frac{3\pi}{4} \Rightarrow \cos\varphi = \frac{-3\pi}{4} \div \frac{\pi}{2}$$

$$\varphi = \cos^{-1} -\frac{3}{2} \neq$$

$\varphi_0 \Rightarrow$ nulls $\quad \varphi_0 = \pm 60^\circ$

Example (2)

$d = \lambda/2$

viii) Hp → direction.

$$\cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm(2k+1)\frac{\pi}{4}$$

$k=0$

$$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm\frac{\pi}{4}$$

$$\frac{\pi}{2}\cos\phi = \pm\left(\frac{\pi}{4} - \frac{\pi}{4}\right) \qquad \phi = \cos^{-1} 0 \qquad \phi = 90^\circ, 270^\circ$$

$k=1$

$$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm(3)\frac{\pi}{4}$$

$$\frac{\pi}{2}\cos\phi = \pm\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)$$

$$\frac{\pi}{2}\cos\phi = \pm\frac{\pi}{2} \qquad \phi = \cos^{-1} \pm 1 \qquad \phi = 0, 180^\circ$$

Hp directions $\phi = 0, 90^\circ, 180^\circ, 270^\circ$

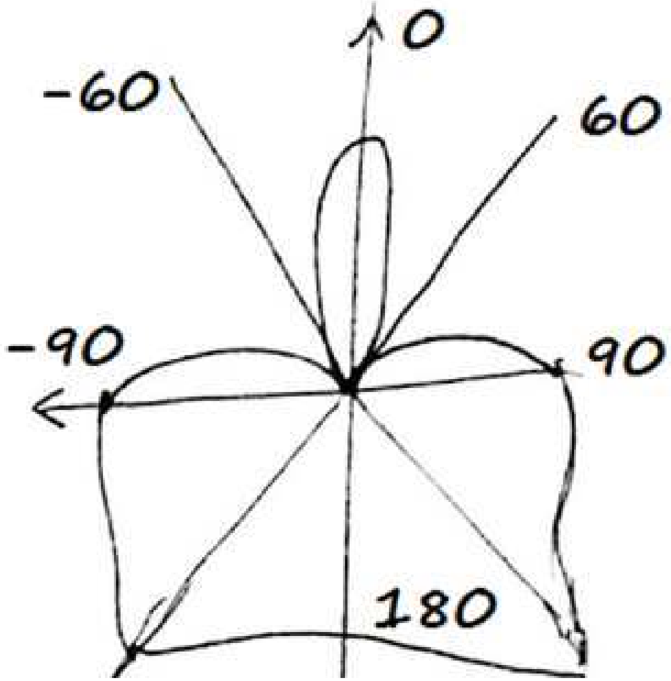
$\downarrow$ $k=1$, $\downarrow$ $k=0$, $\downarrow$ $k=1$, $\downarrow$ $k=0$

Example (2)

1. <u>max</u> $\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{4}\right) = \pm k\pi \rightarrow k=0 \quad \theta = \pm 120$

2. <u>nulls</u> $\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{4}\right) = \pm(2k+1)\frac{\pi}{2} \rightarrow k=0 \quad \theta = \pm 60$

3. <u>Half power</u> $\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{4}\right) = \pm(2k+1)\frac{\pi}{4}$
   - $\rightarrow k=0 \quad (90, 180, 270)$
   - $\rightarrow k=1 \quad (\theta = 0)$



Example (3)

let: $d = \frac{\lambda}{4}$

$$E = \cos\left(\frac{\pi}{4}\cos\varphi + \frac{\pi}{4}\right)$$

i) max

$$\cos\left(\frac{\pi}{4}\cos\varphi + \frac{\pi}{4}\right) =$$

$$\frac{\pi}{4}\cos\varphi + \frac{\pi}{4} = \pm k\pi$$

$k = 0$

$$\frac{\pi}{4}\cos\varphi = -\frac{\pi}{4}$$

$$\varphi = \cos^{-1} -1$$

$$\varphi_m = 180^\circ$$

ii) null

$$\frac{\pi}{4}\cos\varphi + \frac{\pi}{4} = \pm(2k+1)\frac{\pi}{2}$$

$k = 0$

$$\frac{\pi}{4}\cos\varphi + \frac{\pi}{4} = \pm\frac{\pi}{2}$$

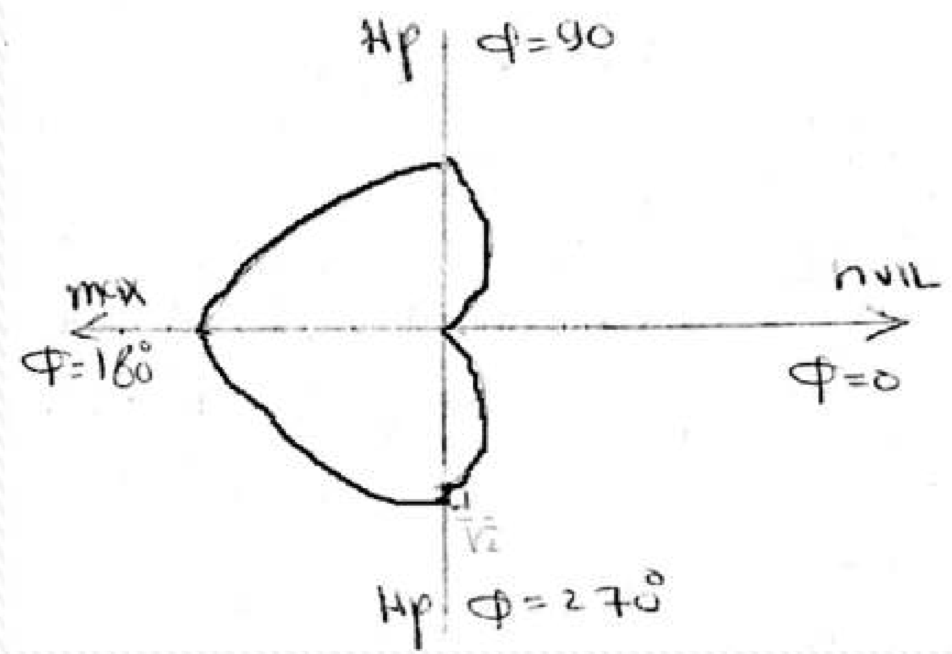
$$\frac{\pi}{4}\cos\varphi = \frac{\pi}{2} - \frac{\pi}{4} \qquad \varphi = 0$$

Example (3)

iii) Hp $\frac{\pi}{4}\cos\phi + \frac{\pi}{4} = \pm(2k+1)\frac{\pi}{4}$

$K=0$ $\frac{\pi}{4}\cos\phi = +\frac{\pi}{4} - \frac{\pi}{4}$ $\cos\phi = 0$ $\phi = 90^\circ, 270^\circ$

$\frac{\pi}{4}\cos\phi = -\frac{\pi}{4} - \frac{\pi}{4}$ $\cos^{-1}(-2)$ ✗



Case (4) : Same amplitude and any phase

$$\psi = d_r \cos \phi + \delta$$

$$E = E_0(e^{j\psi/2} + e^{-j\psi/2}) = 2E_0 \cos \frac{\psi}{2}$$

$$E = \cos \frac{\psi}{2}$$

Case (5) : Unequal amplitude and any phase

Case 5

2 Isotropic Point sources.

if amp (not equal) Phase not equal.

- unequal amp and any Phase difference.

- any "φ" phase difference δ

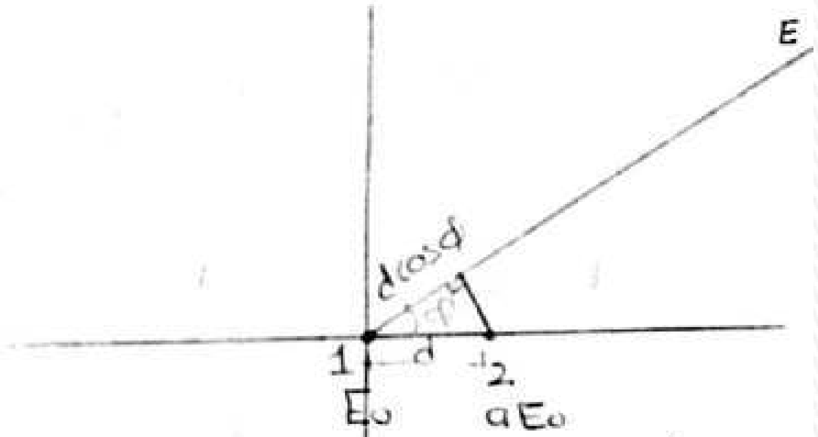
- unequal amp

Source 1 : E_0 : Amp

Source 2 : aE_0 : Amp

(*) E_0 (E_1) : Phase = 0

(*) E_2 : angle ψ



a: Fraction From 0 → 1

$0 \leq a \leq 1$

Case (5) : Unequal amplitude and any phase

$$|E| = \sqrt{(aE_o \sin\psi)^2 + (E_o + aE_o \cos\psi)^2}$$

magnitude

$$E_o \sqrt{(1 + a\cos\psi)^2 + (a\sin\psi)^2}$$

$$\psi = \beta d \cos\phi + \delta$$

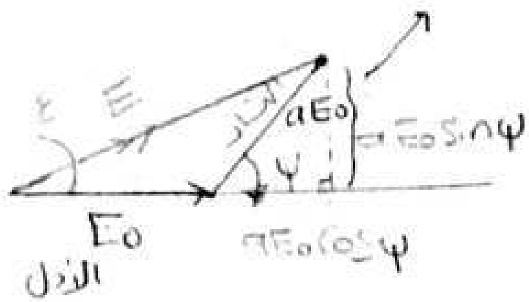
→ phase angle

بين E_o و الأفقي.

angle of electric field :-

$$\text{angle} = \arctan = \tan^{-1}\left[\frac{a\sin\psi}{1 + a\cos\psi}\right]$$

ايجاد ال magnitude



Non isotropic point sources but similar Pattern Multiplication

Non isotropic point sources but similar

If each source had field pattern of :

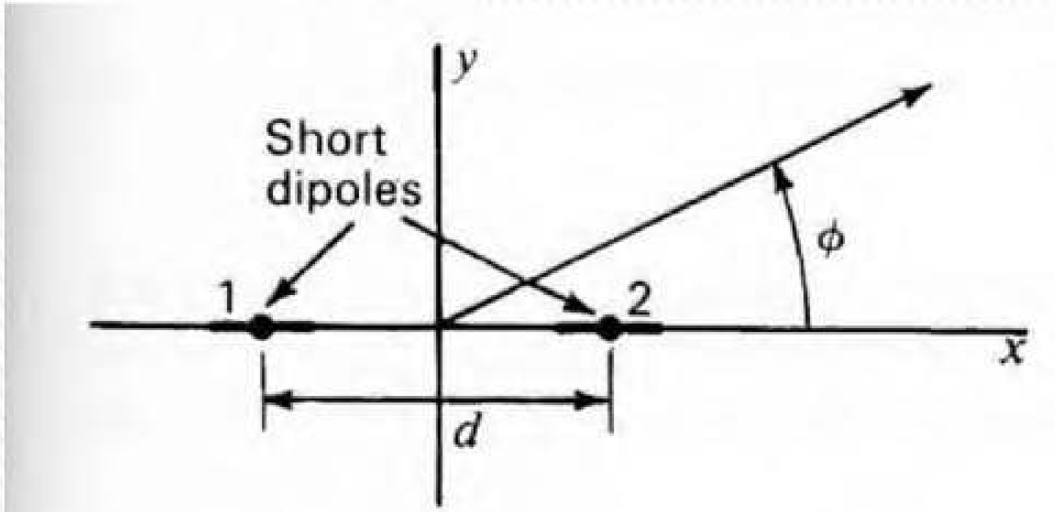
$$E_0 = E_0' \sin \phi$$

the field pattern of the array

$$E = \sin \phi \cos \frac{\psi}{2}$$

where $\psi = d_r \cos \phi + \delta$

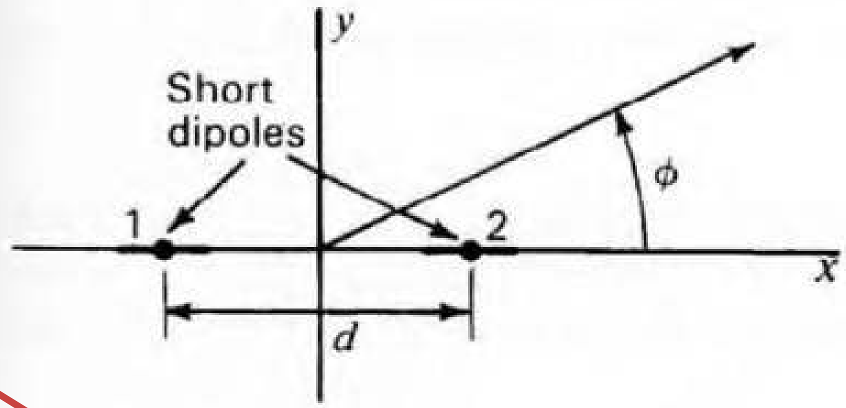
Note: Sources on or parallel to X axis



Example (4)

Example Assume two identical point sources separated by a distance d, each source having the field pattern given by (1) as might be obtained by two short dipoles arranged as in Fig. Let $d = \lambda/2$ and the phase angle $\delta = 0$. Then the total field pattern is

$$E = \sin\phi \cos\left(\frac{\pi}{2}\cos\phi\right)$$

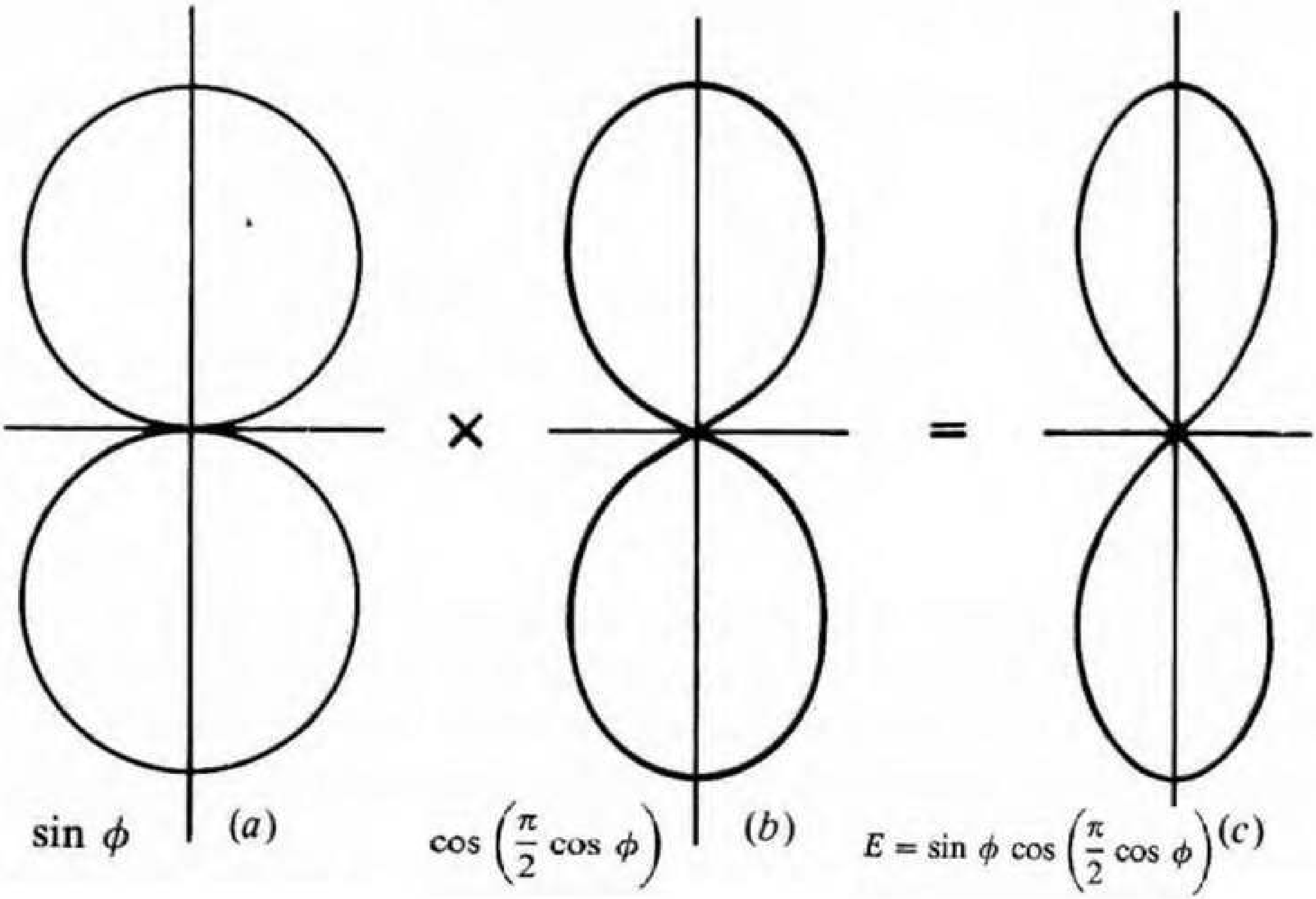


Individual pattern

Array pattern

Total Array Pattern

Non isotropic point sources but similar



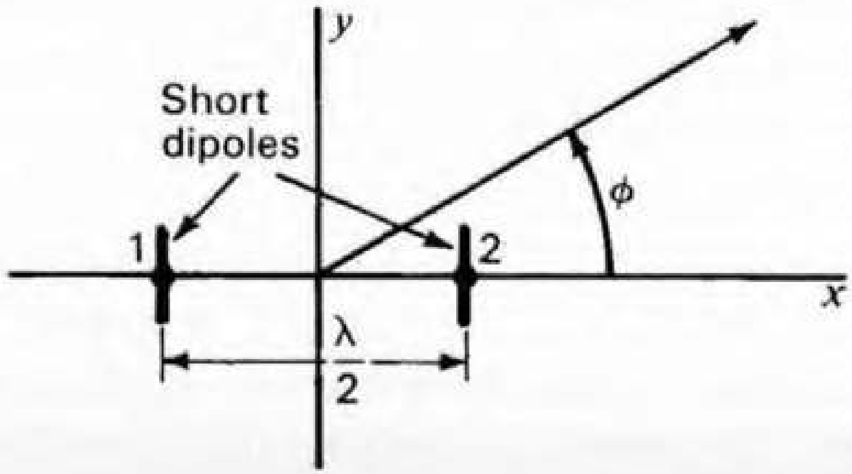
Example (5)

Example Let us consider next the situation in which $d = \lambda/2$ and $\delta = 0$ as in Example 1 but with individual source patterns given by

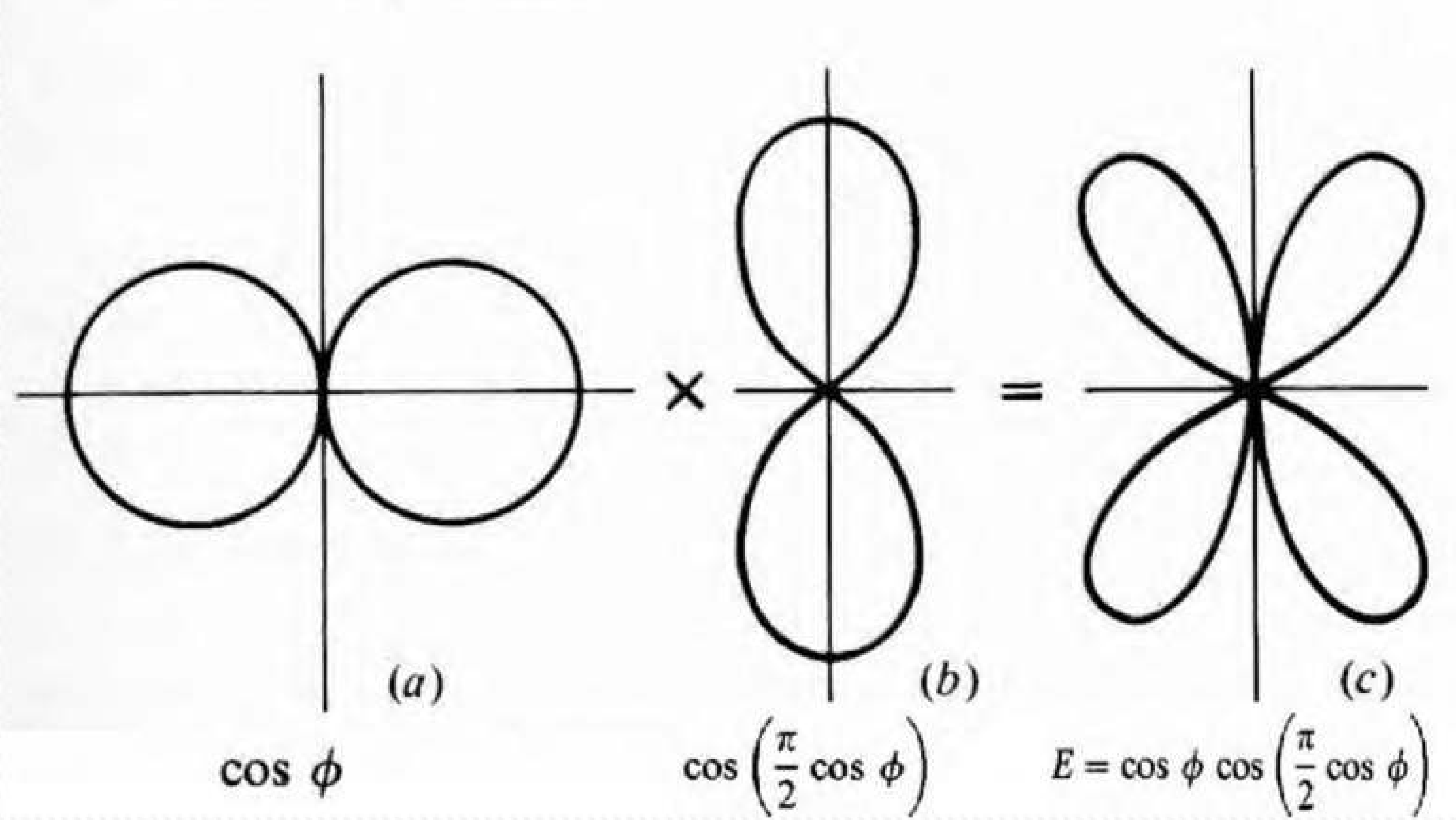
$$E_0 = E_0' \cos \phi$$

This type of pattern might be produced by short dipoles oriented parallel to the y axis as in Fig. Here the maximum field of the individual source is in the direction ($\phi = 0$) of a null from the array, while the individual source has a null in the direction ($\phi = 90°$) of the pattern maximum of the array. By the principle of pattern multiplication the total normalized field is

$$E = \cos \phi \cos \left(\frac{\pi}{2} \cos \phi \right)$$

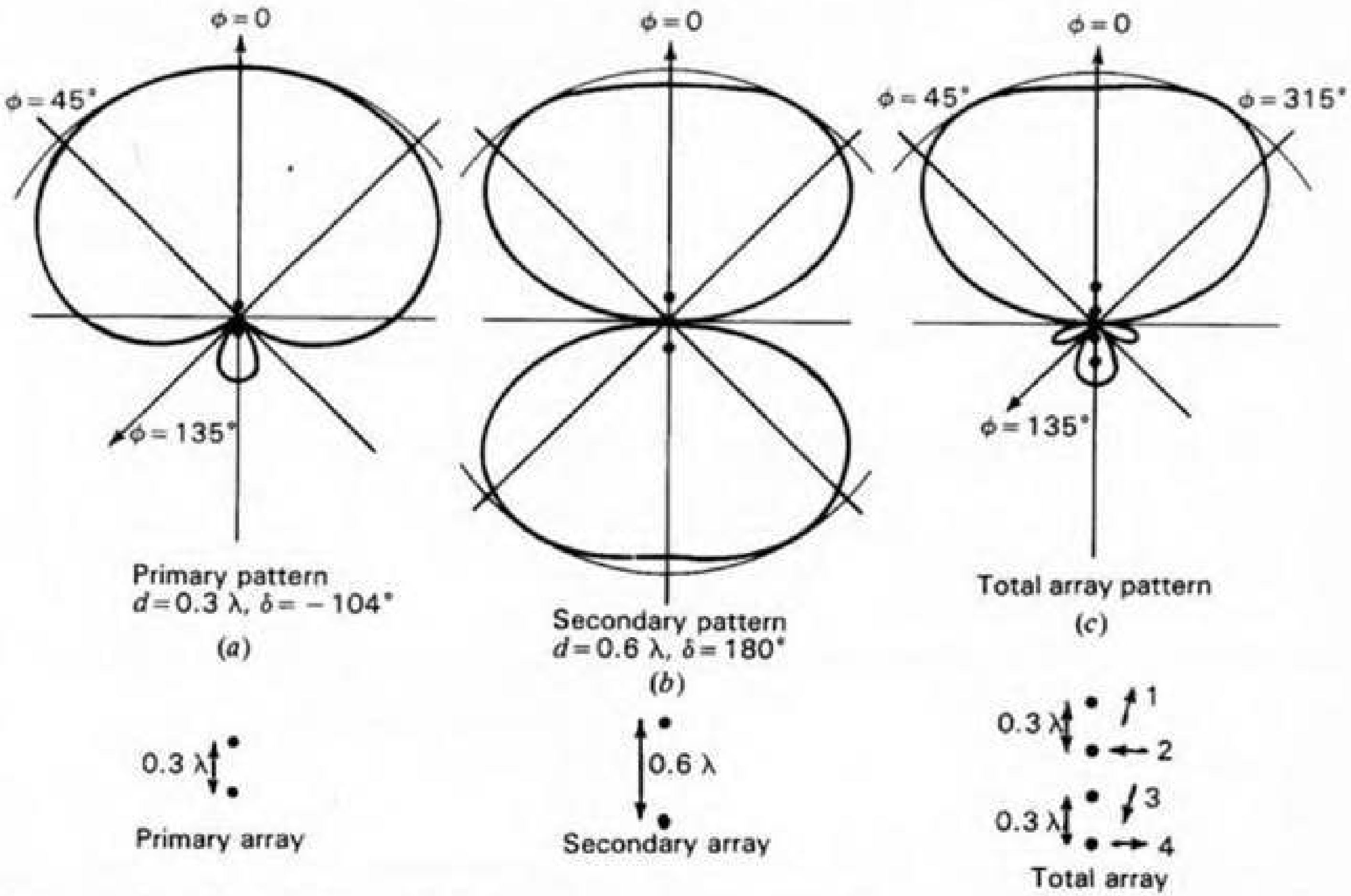


Example (5)


×
=
(a)
(b)
(c)
cos ϕ
cos $\left(\frac{\pi}{2}\cos\phi\right)$
$E = \cos\phi\cos\left(\frac{\pi}{2}\cos\phi\right)$

Example (6)

$$E = \cos(54^\circ \cos\phi - 52^\circ)\cos(108^\circ \cos\phi + 90^\circ)$$



Field patterns of primary and secondary arrays of 2 isotropic sources which multiplied together give pattern of total array of 4 isotropic sources.

Next Lecture (6)

Chapter(4): Arrays of point Sources

N - Element Array (isotropic/non-isotropic)

(End fire – Broadside)

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Thank You